

C&EE 141

Beam-Columns

# Beam Columns

- Most structural elements are subjected to both bending and axial loads (compression or tension).
- In many cases, it is reasonable to neglect combined effects:
  - Pin-pin beam with very minor axial forces
  - Pin-pin column with only accidental eccentricities
- However, many members have significant amounts of both bending and axial load, and must be designed as “beam-columns”.

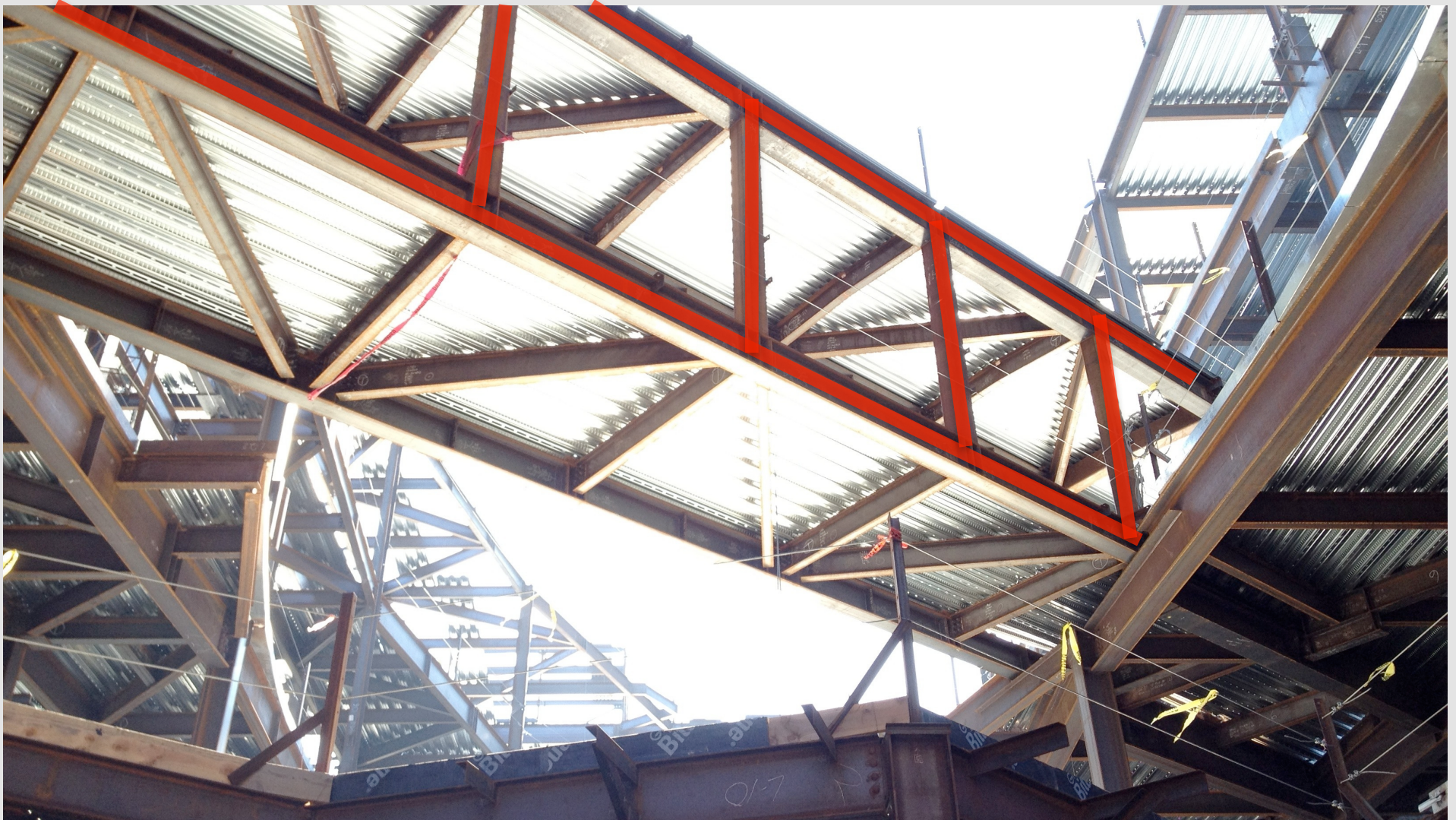
# Examples of Beam-Columns

- Beams in braced frames
- “Drag” and “Chord” members
- Beams and columns in moment-resisting frames
- Columns that brace exterior wall construction for wind & seismic loading





# Beam-Column Examples





# Beam-Column Examples

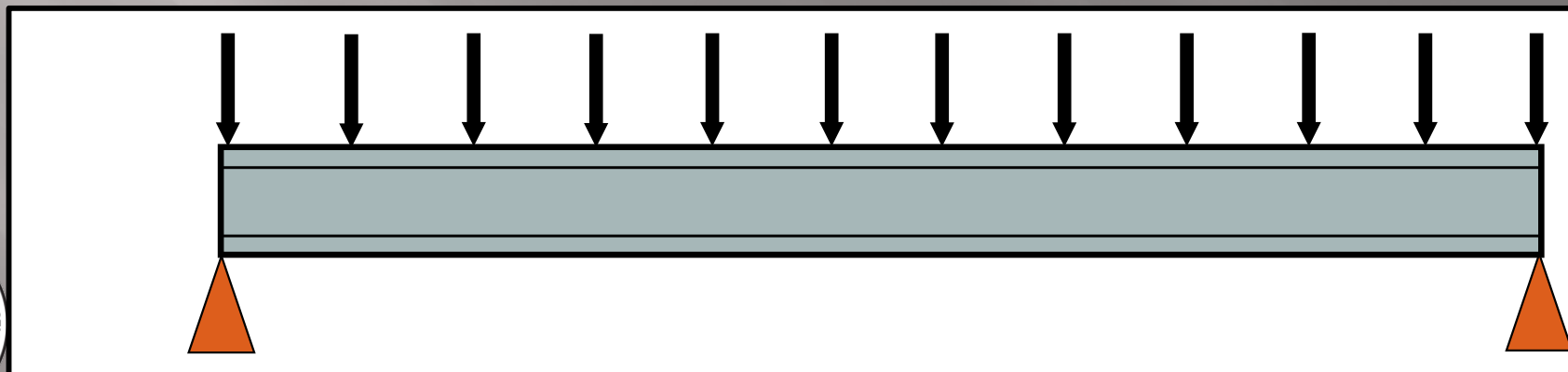


# Second Order Analysis

# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Initially, consider a member subjected to flexure only.





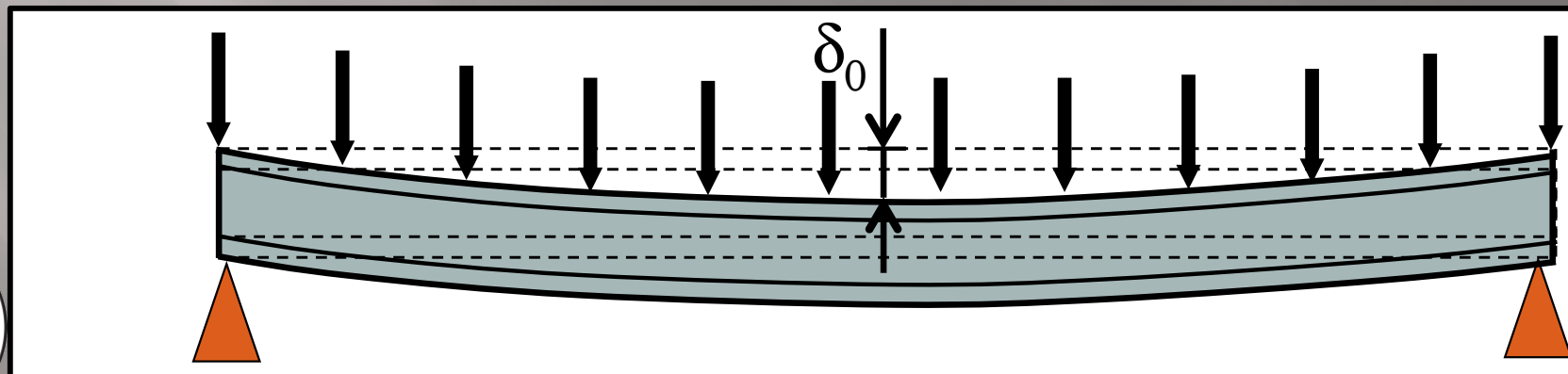
# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Initially, consider a member subjected to flexure only.

Application of load results in mid-span deflection  $\delta_0$ ,

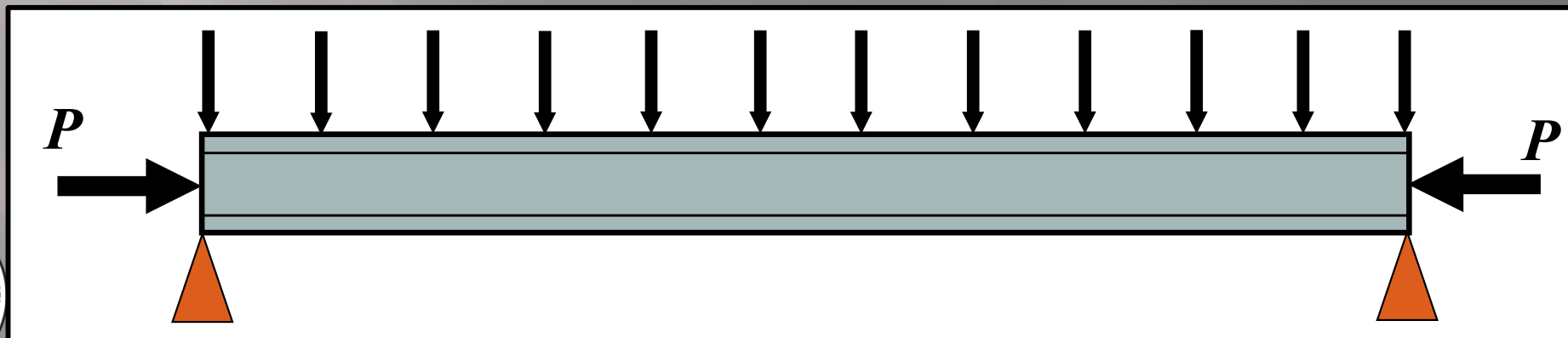
$$\delta_0 = \frac{5}{384} \frac{w L^4}{EI} \quad \text{from basic derivations.}$$



# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

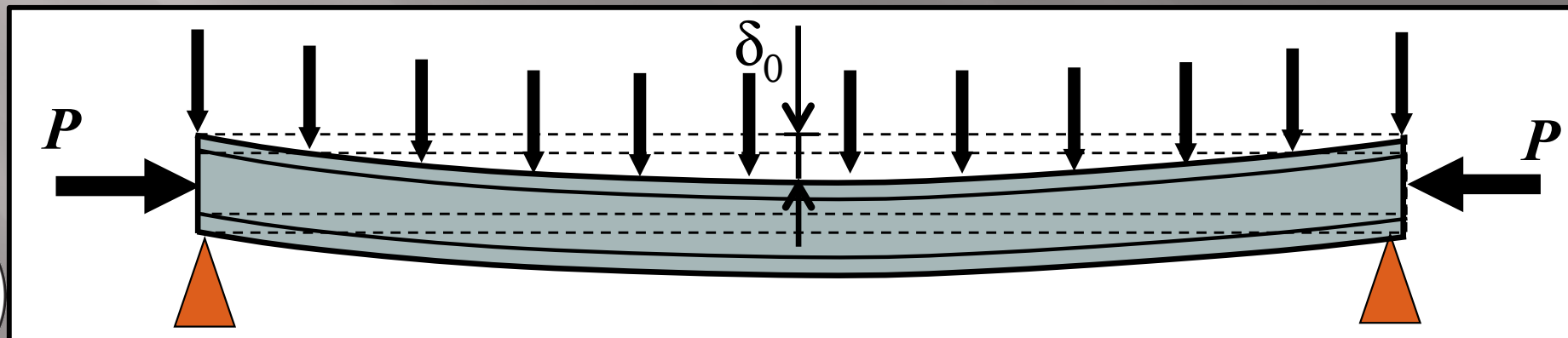
Now consider the same member with Axial load  $P$ .



# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment  $P(\delta_0)$  at center of span.





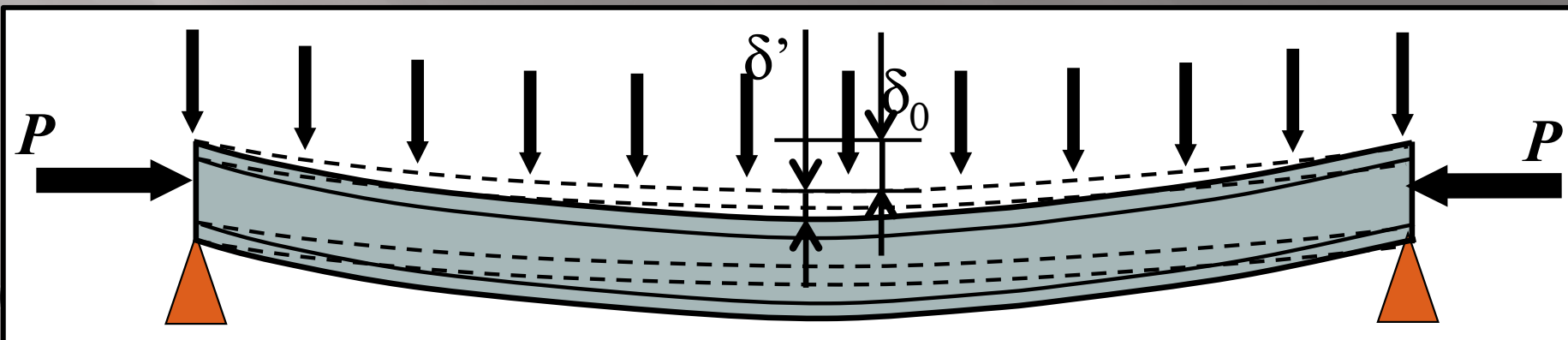
# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment  $P(\delta_0)$  at center span.



Additional moment then results in displacement  $\delta'$ .



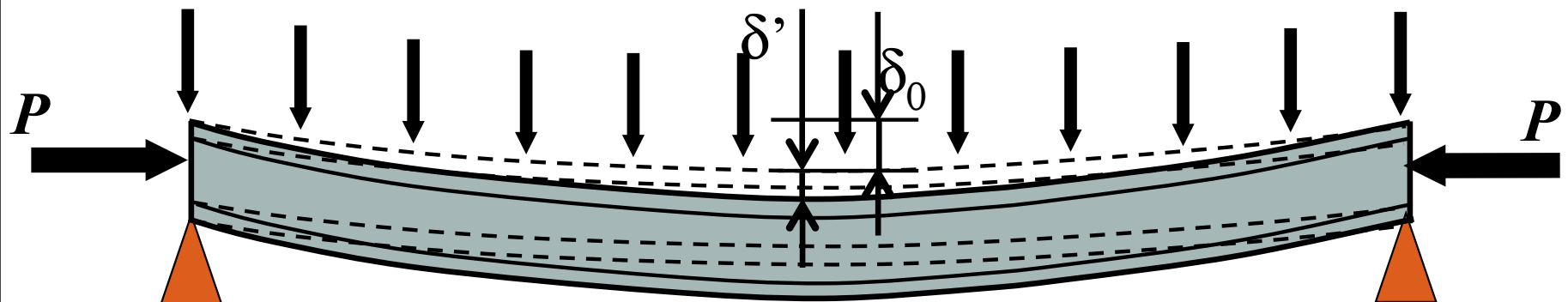
# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment  $P(\delta_0)$  at center span.

Additional moment then results in displacement  $\delta'$ .

Resulting in additional moment  $P(\delta')$



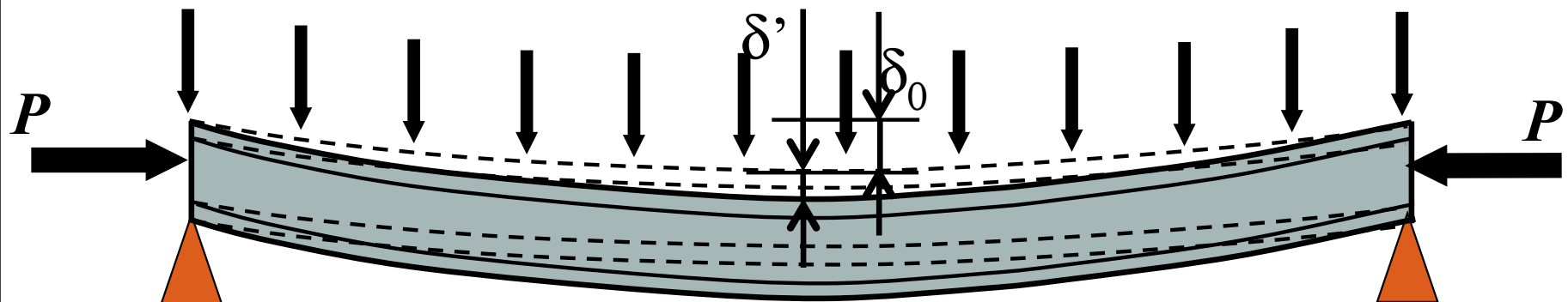
# Second Order Effects

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Axial force acting through deformations results in additional moment  $P(\delta_0)$  at center span.

Additional moment then results in displacement  $\delta'$ .

Resulting in additional moment  $P(\delta')$





# Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

This either results in an incremental failure, or stabilizes.

$$\delta > \delta_0 \text{ and } M > M_0$$

where  $\delta_0$  and  $M_0$  are first order results based on original geometry.

$$M = M_0 + P\delta$$

but  $\delta$  depends on  $M...$



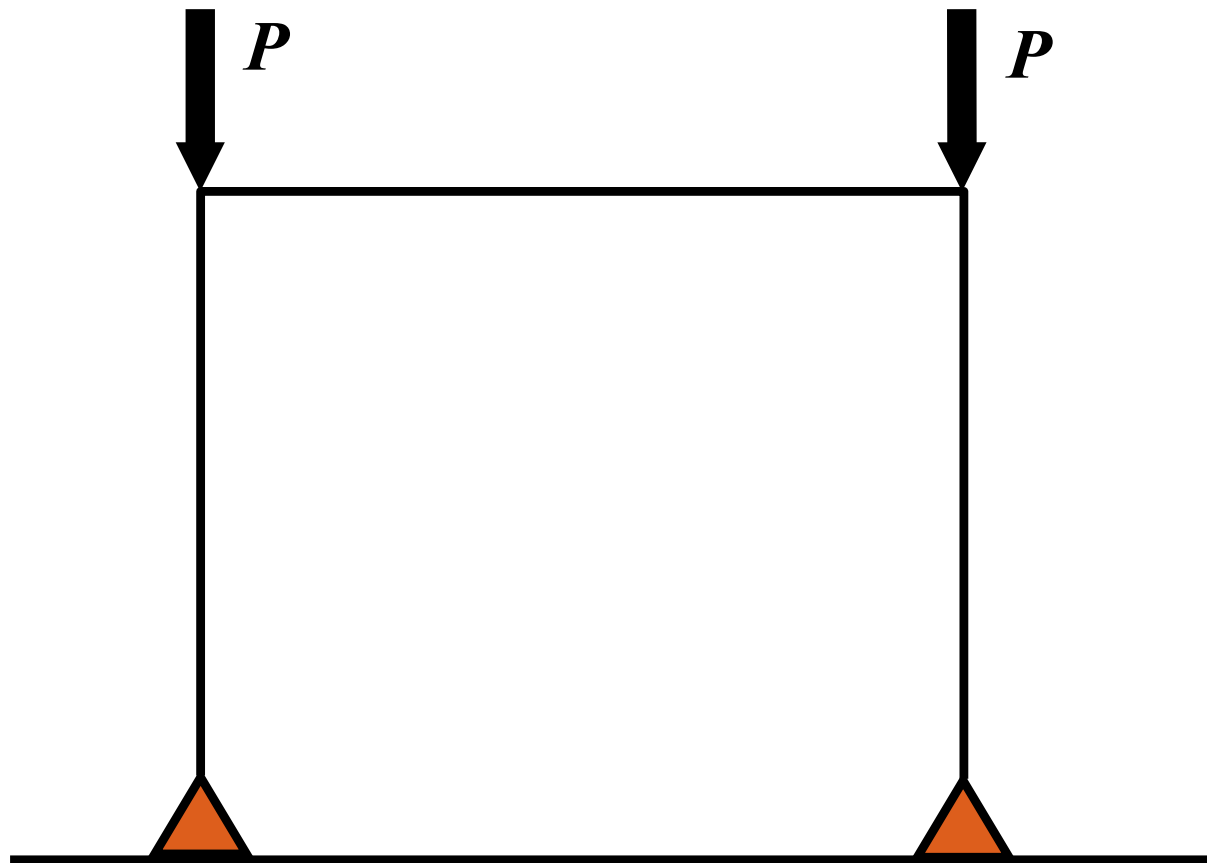
# Second Order Effects

When considering a frame with loads applied at joints the same principles can be applied.

In these cases, we define joint deflections as  $\Delta$ , with  $\Delta_0$  being the first order joint deflection.

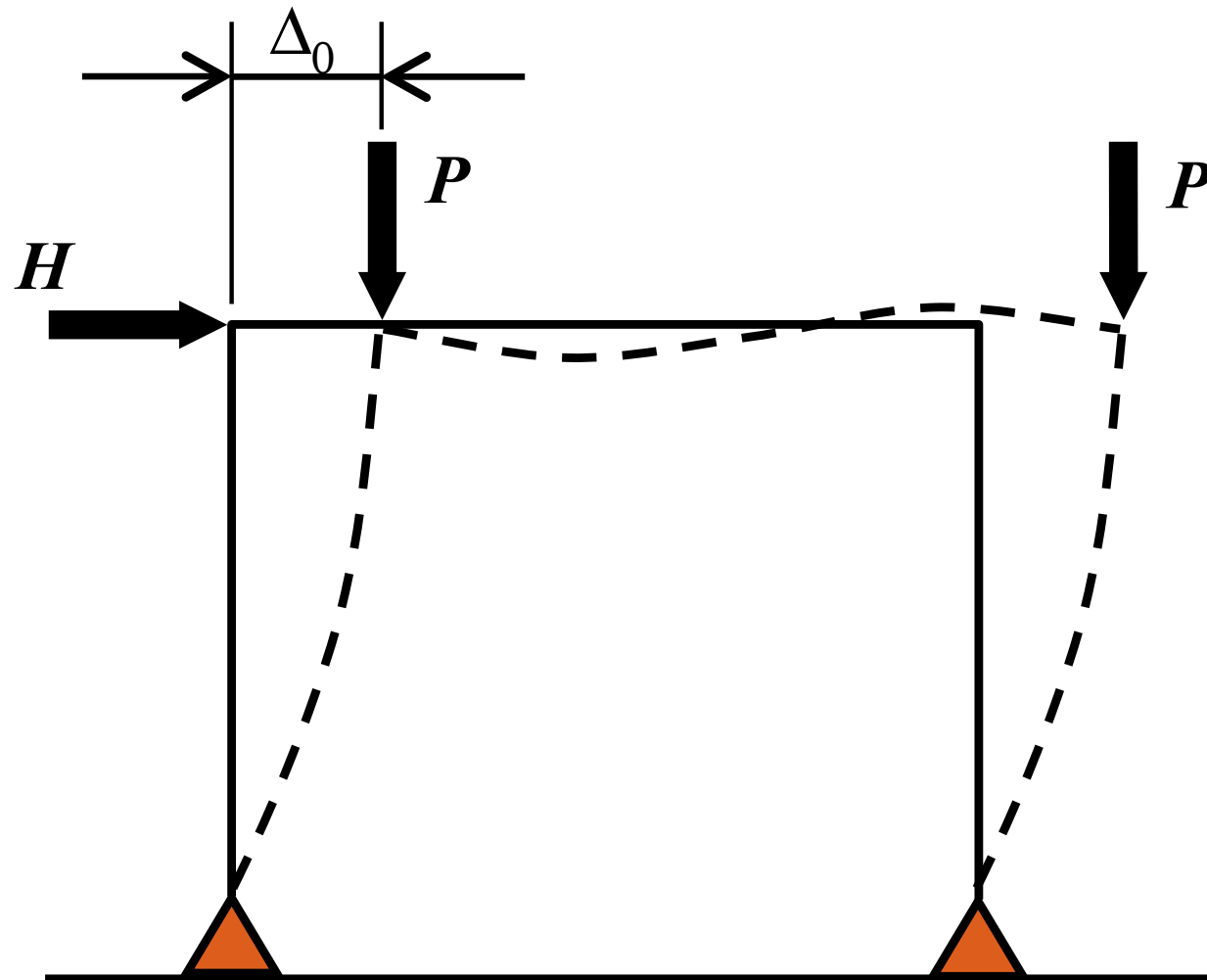


# Second Order Effects

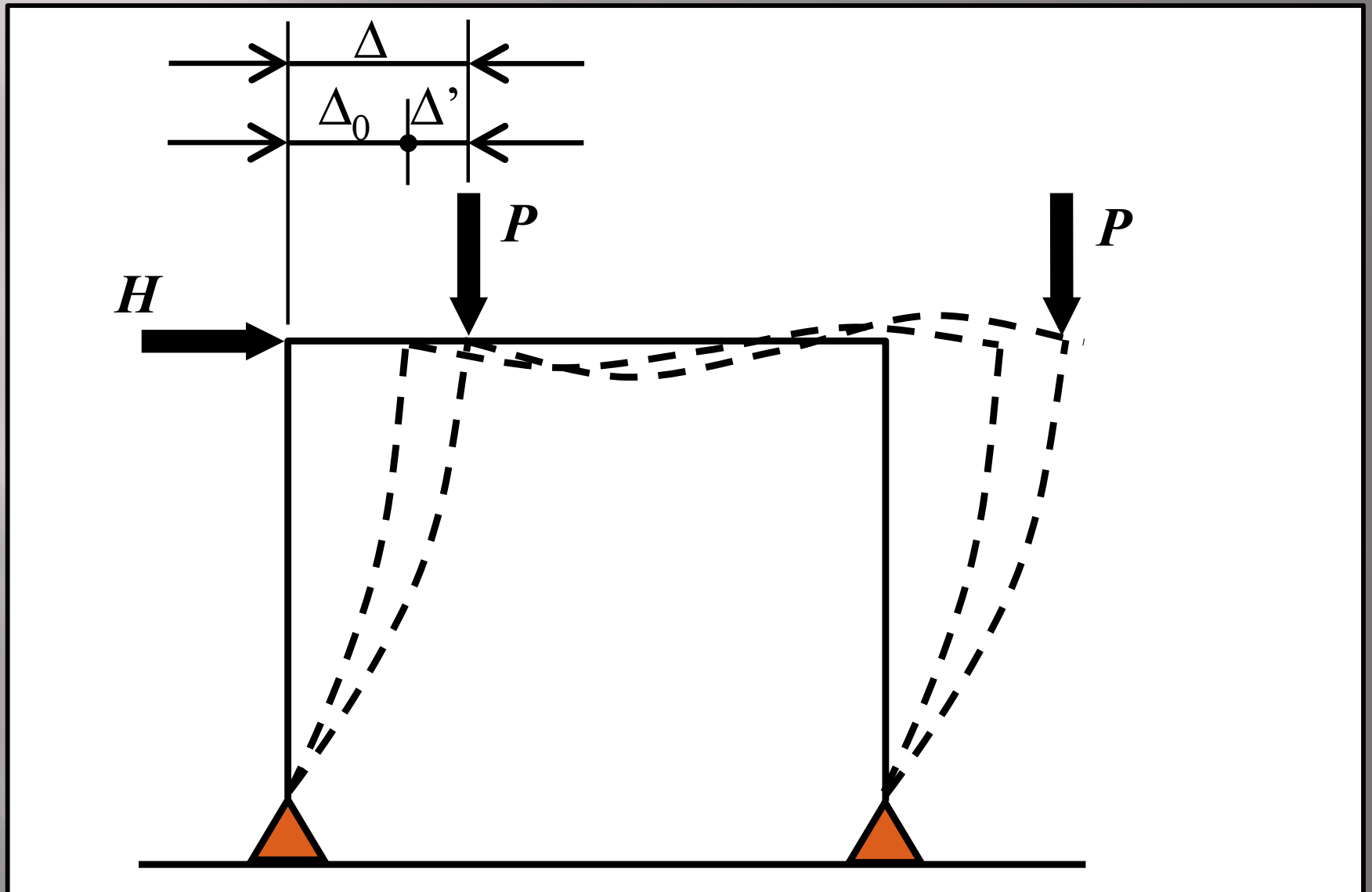




# Second Order Effects

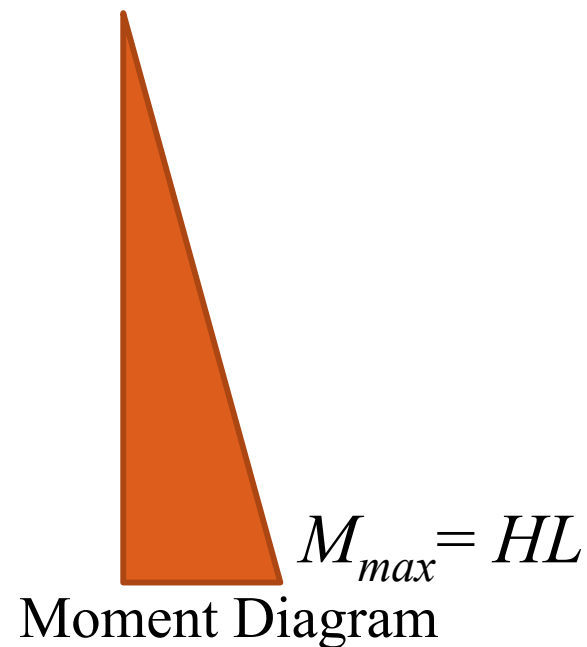
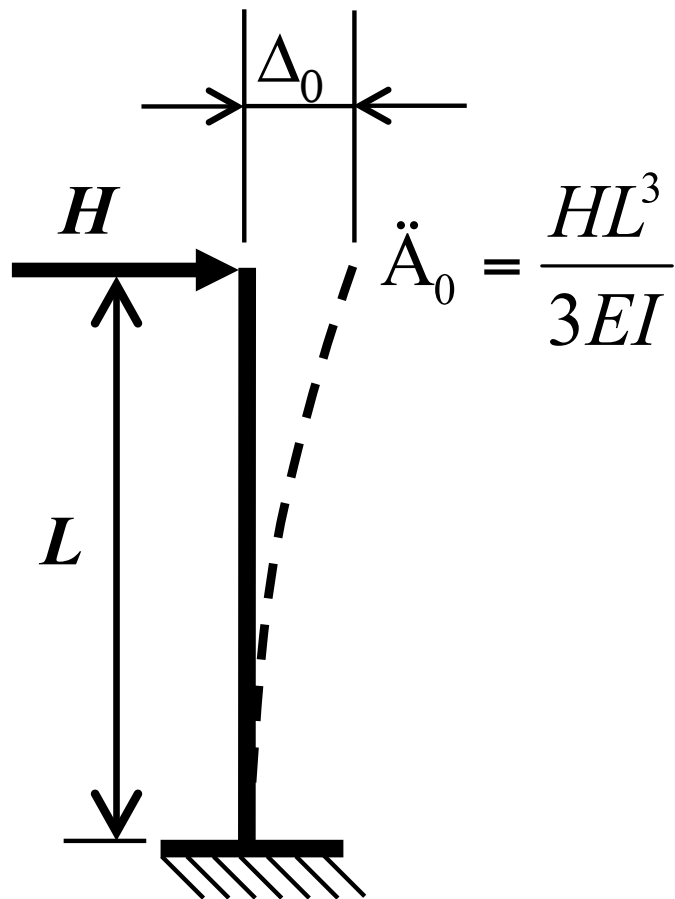


# Second Order Effects



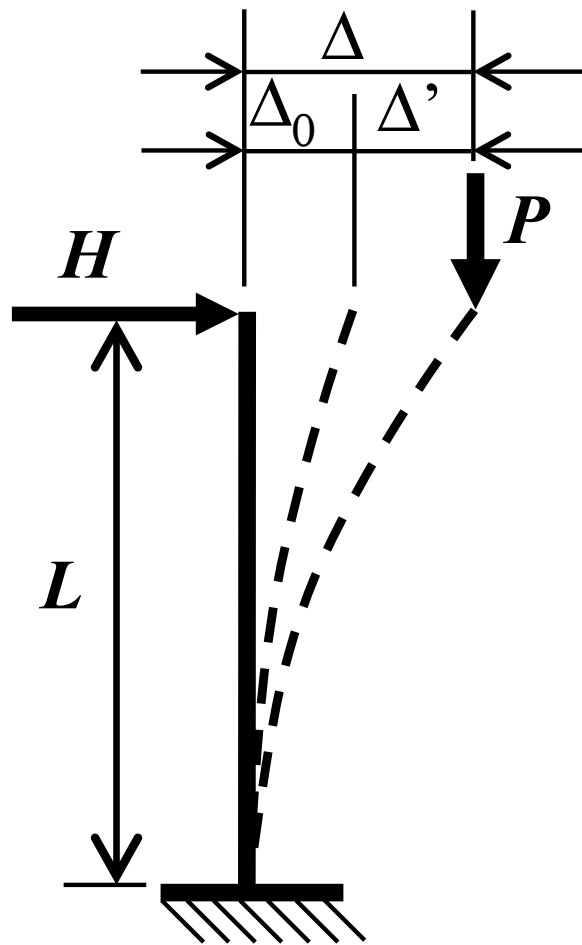
# Second Order Effects

Assuming deflections are in the shape of a sine curve



# Second Order Effects

Assuming deflections are in the shape of a sine curve



Moment Diagram  $M_{max} = HL + P\Delta$

# Second Order Effects

## Tension forces and Flexure

Tension forces on a member tend to “straighten” the member. Usually, they do not introduce 2<sup>nd</sup> order effects.

Multiple states of stress are still present and need to be accounted for.

Tension in a member can also make lateral torsional buckling less likely to occur.





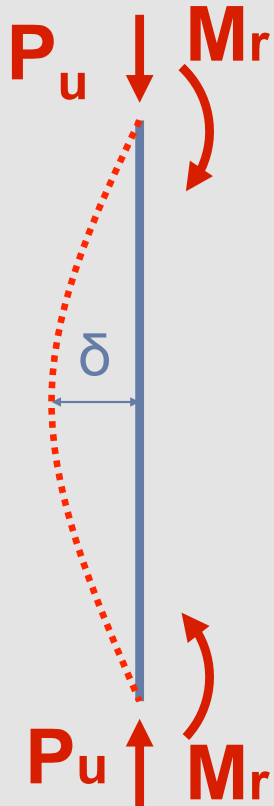
# Approximate 2<sup>nd</sup> Order Analysis

- Per AISC 360-10 allows several methods to be used as alternatives to more rigorous (and tedious) 2<sup>nd</sup> Order Analysis
- Common procedures:
  - “Direct Analysis Method” (DM) per AISC Chapter C.2
  - Approximate 2<sup>nd</sup> Order Analysis per AISC Appendix 8

# Approximate 2<sup>nd</sup> Order Analysis

- AISC Spec Appendix 8, Approximate 2<sup>nd</sup> Order Analysis
- Common in local practice
- Consists of 1<sup>st</sup> order elastic analysis with multipliers  $B_1$  and  $B_2$  applied to resulting moments (“Moment Magnification”)

# Moment Magnification of Columns with No Sidesway (Braced Frame)

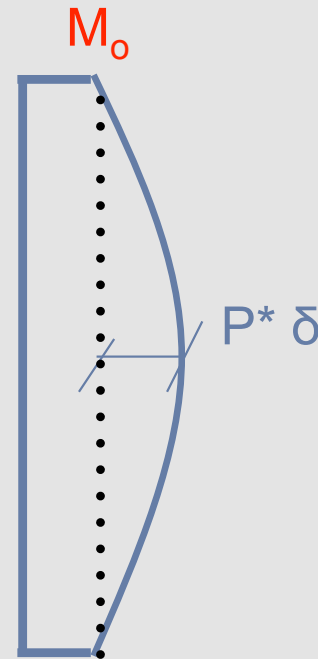
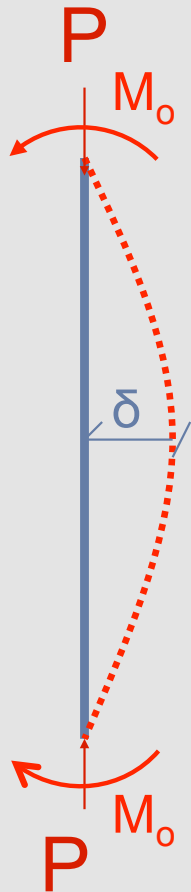


$$M_r = M_{nt} + P_u \delta = B_1 M_{nt}$$

2<sup>nd</sup> Order Moment  
1<sup>st</sup> Order Moment

“Member Effect” per Geschwindner

# $B_1$ Multiplier For No Sidesway

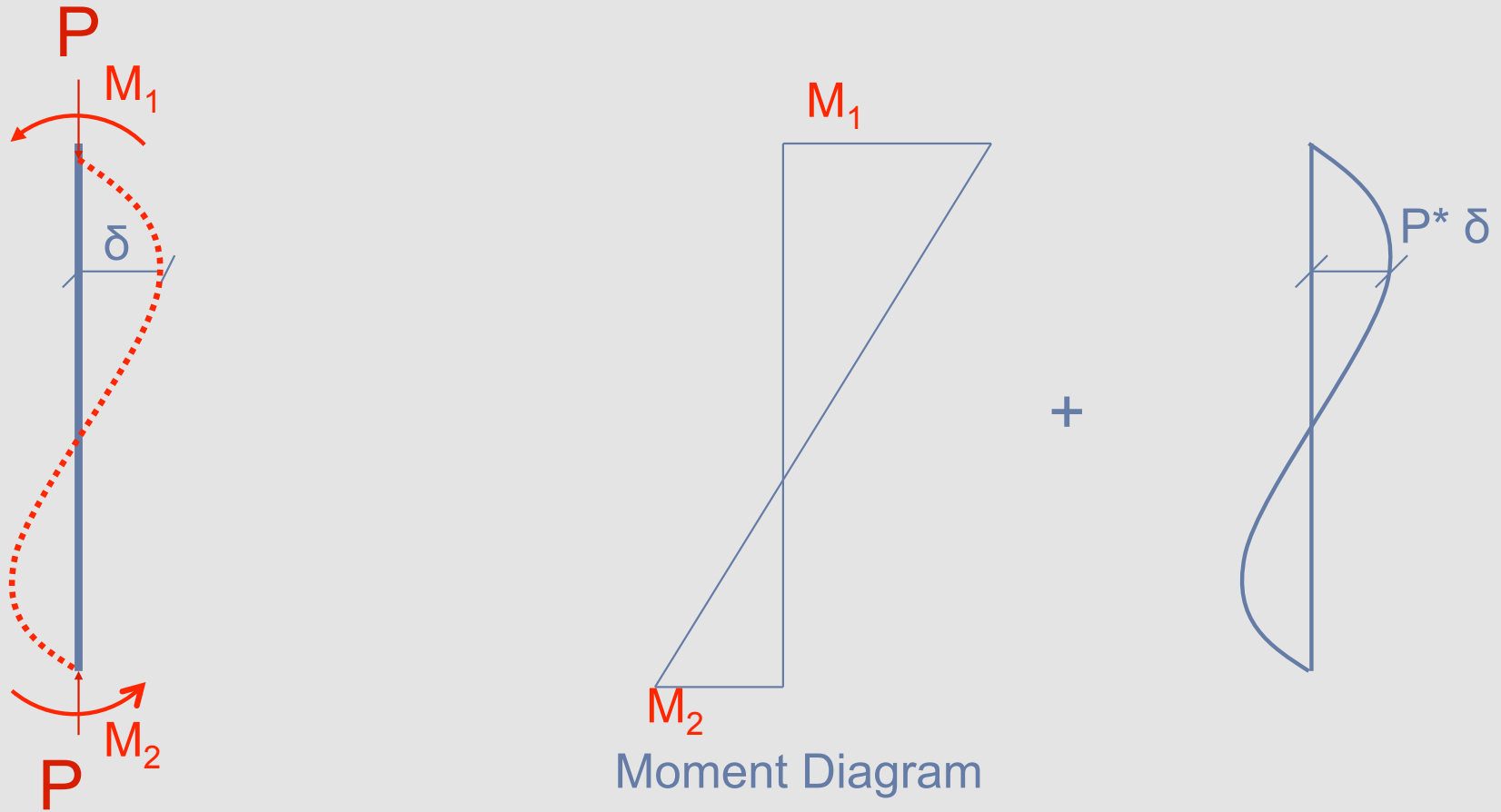


$$M_{\max} = M_o + P^* \delta$$

Moment Diagram

End moments in opposing direction: single curvature bending.  
Primary and secondary moments are additive.

# $B_1$ Multiplier For No Sidesway



End moments in same direction: double curvature bending.  
Depending on magnitude of  $P$ ,  $M_1$ ,  $M_2$ , and  $L$ ,  $M_{\max}$  may be equal to  $M_2$  or greater than  $M_2$ .

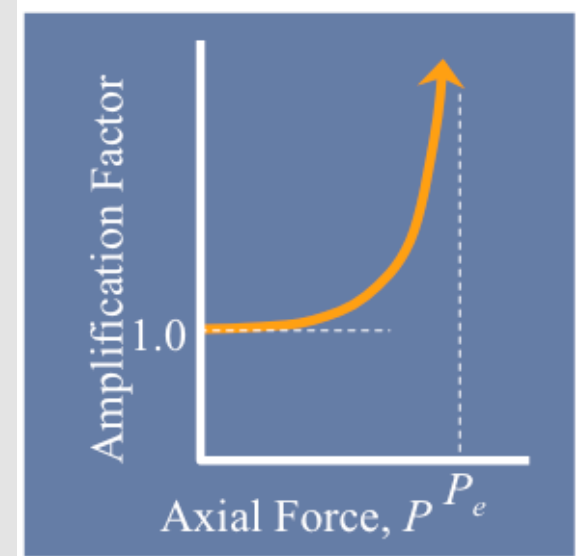


# $B_1$ Multiplier For No Sidesway





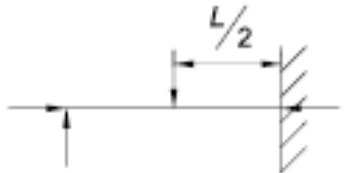

- The worst case will always be single curvature bending; double curvature bending tends to minimize the moment amplification.
- This effect is accounted for with a  $C_m$  factor.

# $B_1$ Multiplier For No Sidesway

- $B_1 = C_m / (1 - \alpha P_r / P_{e1}) \geq 1$ 
  - $C_m$  will always be  $\leq 1.0$
  - $\alpha = 1.0$  for LRFD
- $P_{e1} = \pi^2 EA_g / (KL/r)^2$ 
  - Computed about the axis of bending.
- $C_m = 0.6 - 0.4(M_1/M_2)$ 
  - For no transverse loads acting on the member
  - $M_1$ =Smaller end moment;  $M_2$ =Larger end moment
  - $M_1/M_2 > 0$  for reverse curvature,  $M_1/M_2 < 0$  for single curvature
- See Table C-A-8.1 in AISC 360-10 for  $C_m$  factors when beam-column is loaded between joints.

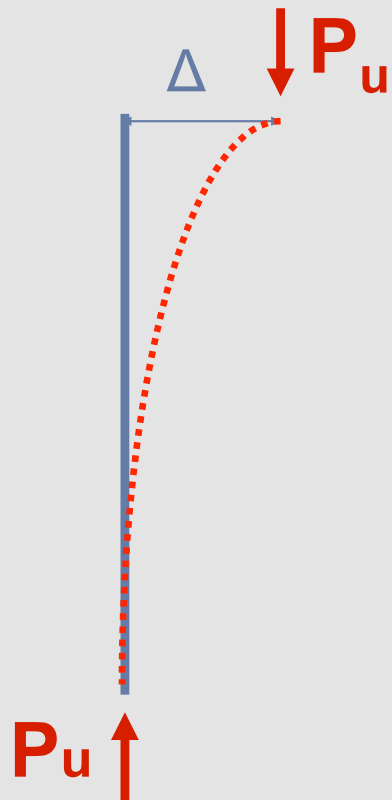


**TABLE C-A-8.1**  
**Amplification Factors  $\psi$  and  $C_m$**

Case	$\psi$	$C_m$
	0	1.0
	-0.4	$1 - 0.4 \frac{\alpha P_r}{P_{e1}}$
	-0.4	$1 - 0.4 \frac{\alpha P_r}{P_{e1}}$
	-0.2	$1 - 0.2 \frac{\alpha P_r}{P_{e1}}$
	-0.3	$1 - 0.3 \frac{\alpha P_r}{P_{e1}}$
	-0.2	$1 - 0.2 \frac{\alpha P_r}{P_{e1}}$

# Moment and Axial Magnification of Columns with Sidesway (Moment Frames)

“Structure Effect” per Geschwindner



$$M_r = M_{lt} + P_u \Delta = B_2 M_{lt}$$

2<sup>nd</sup> Order Moment

1<sup>st</sup> Order Moment

$$P_r = P_{nt} + B_2 P_{lt}$$

2<sup>nd</sup> Order Axial Load

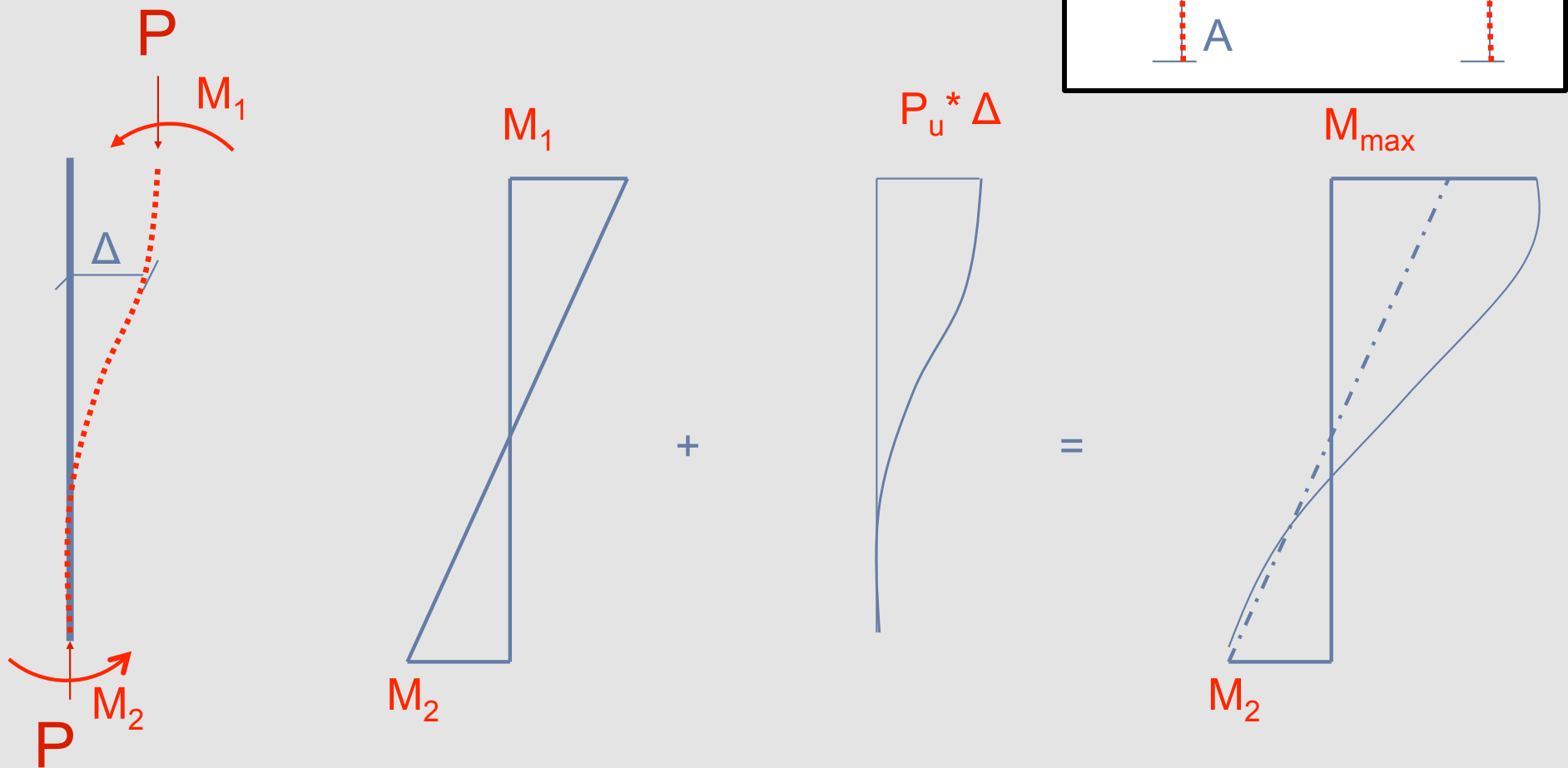
1<sup>st</sup> Order Axial Load

# $B_2$ Multiplier for Sidesway

- In beam-columns whose ends can translate relative to each other:
  - Maximum primary moment due to sidesway is almost always at one end.
  - Maximum secondary moment due to sidesway is always at one end.
  - Amount of sidesway impacts the secondary moment, and is dependent on the deflection of a system of members acting together.



# $B_2$ Multiplier for Sidesway



# $B_2$ Multiplier for Sidesway

- Because 1<sup>st</sup> Order and 2<sup>nd</sup> Order effects are always additive for sidesway, there is no  $C_m$  term in the expression for  $B_2$ .

# $B_2$ Multiplier for Sidesway

- $B_2 = 1/(1 - \alpha \Sigma P_{nt} / \Sigma P_{e2})$  where
  - $\alpha = 1.00$  (LRFD)
  - $\Sigma P_{nt}$  = Sum of all factored loads acting on all columns in the story
  - $\Sigma P_{e2} = \Sigma \{(\pi EI) / (K_2 L)^2\}$  or  $R_M (\Sigma HL / \Delta H)$
- See AISC 360-10 Appendix 8 for all Equations

# Braced vs. Unbraced Frames

- The two different moments need to be split and amplified separately.
- Calculate amplified loads as follows:
  - $M_r = B_1 M_{nt} + B_2 M_{lt}$  (AISC 360-10 Equation A-8-1)
  - $P_r = P_{nt} + B_2 P_{lt}$  (AISC 360-10 Equation A-8-2)
    - $B_1$  = amplification factor for no lateral translation
    - $M_{nt}$  = maximum moment assuming no lateral translation (calculated regardless of braced vs. unbraced)
    - $B_2$  = amplification factor for lateral translation
    - $M_{lt}$  = maximum moment caused by lateral translation (0 if part of a braced frame)

# Interaction Equations



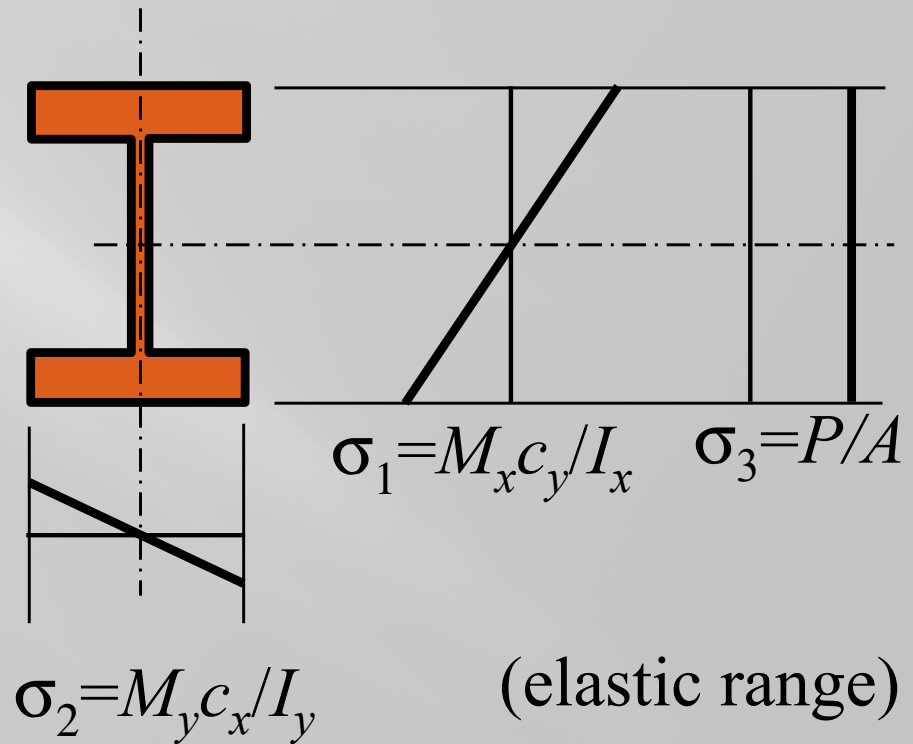
# Interaction Equations

- Each part of a member cross-section has a certain capacity.
- Bending loads “use up” some of the capacity.
- Axial loads “use up” some of the capacity.
- The effects of both bending and axial loads need to be combined in some way.

# Combination of multiple states of stress:

Bending about the major and minor axis will combine to provide maximum stresses in the corner of a W shape.

Axial load will provide uniform stresses across the member and add to other maximum stresses.



# Basic Interaction Equations

- Recall the Basic Design Relationship:
  - Demand  $\leq$  Capacity  
or
  - Demand/Capacity  $\leq 1.0$
- If there is more than one demand to capacity relationship:
  - $\Sigma(\text{Demand} / \text{Capacity}) \leq 1.0$

# Basic Interaction Equations

- For the case of combined bending and axial load:
  - Columns  $P_u \leq \Phi_c P_n$  or  $D/C = P_u / \Phi_c P_n$
  - Beams  $M_u \leq \Phi_b M_n$  or  $D/C = M_u / \Phi_b M_n$
- Basic interaction formula if bending is about 1 axis:
  - $P_u / \Phi_c P_n + M_u / \Phi_b M_n \leq 1.0$
- Basic interaction formula if bending is about 2 axes:
  - $P_u / \Phi_c P_n + M_{ux} / \Phi_b M_{nx} + M_{uy} / \Phi_b M_{ny} \leq 1.0$

# AISC Interaction Equations

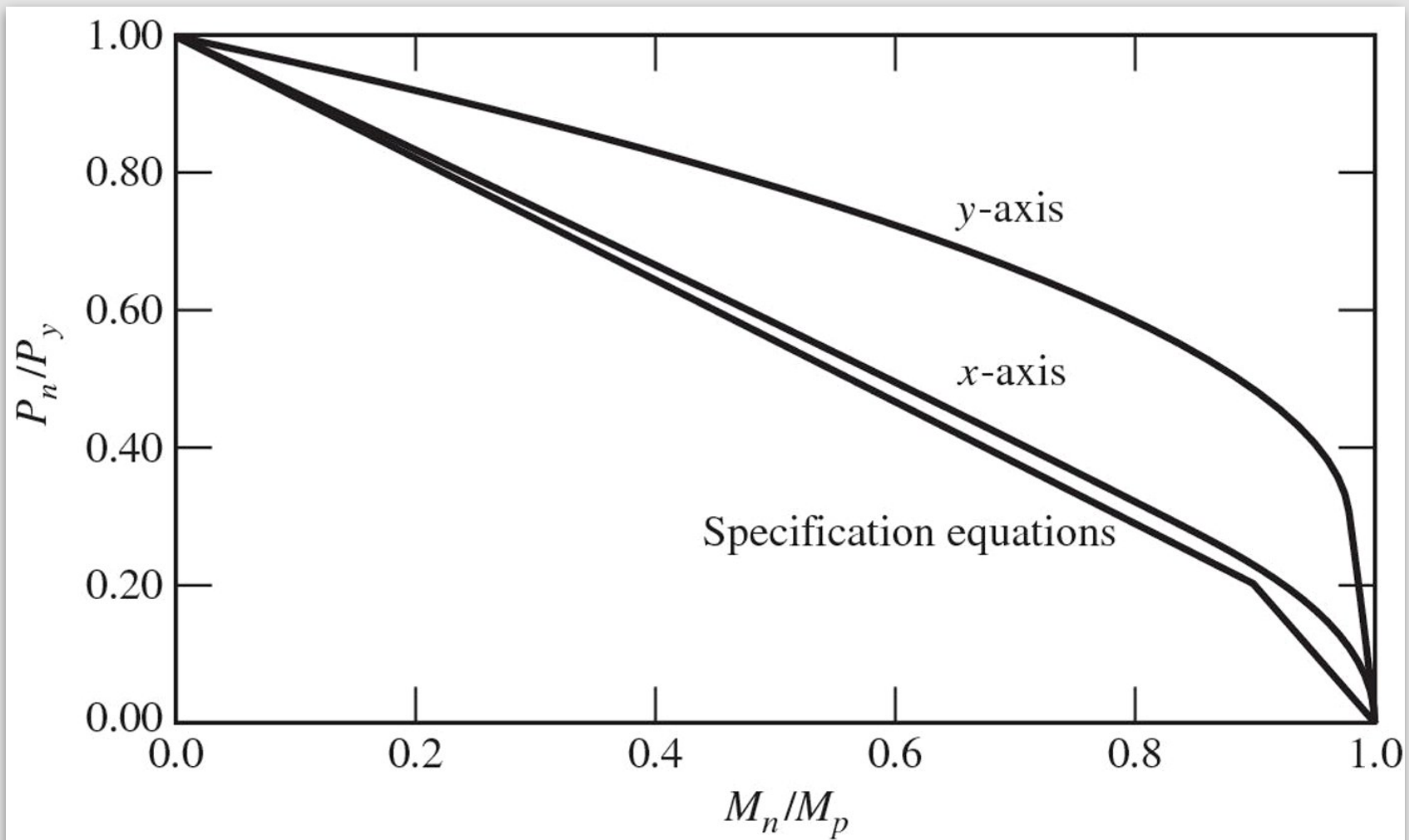


Figure 8.4b  
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# AISC Interaction Equations

- For  $P_r/P_c \geq 0.2$ 
  - $P_r/P_c + (8/9)(M_{rx}/M_{cx} + M_{ry}/M_{cy}) \leq 1.0$
  - AISC 360-10 Equation H1-1a (pg 16.1-73)
  - If axial demand is large, the bending term is slightly reduced.
- For  $P_r/P_c < 0.2$ 
  - $P_r/2P_c + M_{rx}/M_{cx} + M_{ry}/M_{cy} \leq 1.0$
  - AISC 360-10 Equation H1-1b (pg 16.1-73)
  - If axial demand is small, the axial term is reduced.
- Based on matching experimental data

# AISC Interaction Equations for Tension

- $P_r$  = required axial tensile strength
  - Using factored loads
- $P_c$  = available axial tensile strength (capacity)
  - $\phi_t P_n$  where  $\phi_t = 0.9$  or  $0.75$  per Chapter D2
- $M_r$  = required flexural strength
  - Using factored loads
- $M_c$  = available flexural strength (capacity)
  - $\phi_b M_n$  where  $\phi_b = 0.9$

# AISC Interaction Equations for Compression

- $P_r$  = required axial compressive strength
  - Using factored loads
- $P_c$  = available axial compressive strength (capacity)
  - $\phi_c P_n$  where  $\phi_c = 0.9$
  - Use largest slenderness ratio ( $KL/r$ ) for either axis
- $M_r$  = required flexural strength
  - Using factored loads
- $M_c$  = available flexural strength (capacity)
  - $\phi_b M_n$  where  $\phi_b = 0.9$



# Combined Bending & Tension

- For combined compression and flexure
  - If demand/capacity ratios are exceeded then the member is failing in compression in the compression zone of the member.
- For combined tension and flexure
  - If demand/capacity ratios are exceeded then the member is failing in tension in the tension zone of the member.

# AISC Design Tables

# Table 6-1

## Combined Flexure and Compression

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined above.

When  $pP_r \geq 0.2$ :

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \quad (6-1)$$

When  $pP_r < 0.2$ :

$$\frac{1}{2}pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \leq 1.0 \quad (6-2)$$

## Combined Flexure and Tension

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined above.

When  $pP_r \geq 0.2$ :

$$(t_y \text{ or } t_r) P_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \quad (6-3)$$

When  $pP_r < 0.2$ :

$$\frac{1}{2}(t_y \text{ or } t_r) P_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \leq 1.0 \quad (6-4)$$

## Determination of $b_x$ when $C_b > 1.0$

The tabulated values of  $b_x$  assume that  $C_b = 1.0$ . These values may be modified in accordance with AISC *Specification* Sections F1 and H1.2. The following procedure may be used to account for  $C_b > 1.0$ .

$$b_{x(C_b > 1.0)} = \frac{b_{x(C_b = 1.0)}}{C_b} \geq b_{xmin} \quad (6-5)$$

# Table 6-1

	LRFD
Axial Compression	$p = \frac{1}{\phi_c P_n}, (\text{kips})^{-1}$
Strong Axis Bending	$b_x = \frac{8}{9\phi_b M_{nx}}, (\text{kip-ft})^{-1}$
Weak Axis Bending	$b_y = \frac{8}{9\phi_b M_{ny}}, (\text{kip-ft})^{-1}$
Tension Yielding	$t_y = \frac{1}{\phi_t F_y A_g}, (\text{kips})^{-1}$
Tension Rupture	$t_r = \frac{1}{\phi_t F_u (0.75 A_g)}, (\text{kips})^{-1}$

$F_y = 50 \text{ ksi}$ 

Table 6-1 (continued)  
Combined Flexure  
and Axial Force  
W-Shapes



Shape		W24 $\times$											
		103 <sup>c</sup>				94 <sup>c</sup>				84 <sup>c</sup>			
Design		$p \times 10^3$		$b_x \times 10^3$		$p \times 10^3$		$b_x \times 10^3$		$p \times 10^3$		$b_x \times 10^3$	
		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, $KL$ (ft), with respect to least radius of gyration, $r_y$ , or Unbraced Length, $L_b$ (ft), for X-X axis bending	0	1.13	0.753	1.27	0.847	1.26	0.840	1.40	0.933	1.46	0.968	1.59	1.06
	11	1.52	1.01	1.42	0.944	1.67	1.11	1.57	1.05	1.92	1.28	1.80	1.20
	12	1.62	1.08	1.46	0.972	1.78	1.18	1.62	1.08	2.03	1.35	1.87	1.24
	13	1.73	1.15	1.51	1.00	1.90	1.26	1.68	1.12	2.17	1.44	1.93	1.28
	14	1.86	1.23	1.55	1.03	2.04	1.36	1.73	1.15	2.33	1.55	2.00	1.33
	15	2.00	1.33	1.61	1.07	2.21	1.47	1.79	1.19	2.52	1.68	2.08	1.38
	16	2.18	1.45	1.66	1.10	2.40	1.60	1.86	1.24	2.75	1.83	2.16	1.44
	17	2.38	1.58	1.72	1.14	2.62	1.74	1.93	1.28	3.01	2.00	2.25	1.49
	18	2.61	1.74	1.78	1.19	2.88	1.92	2.01	1.33	3.32	2.21	2.34	1.56
	19	2.88	1.92	1.85	1.23	3.18	2.12	2.09	1.39	3.68	2.45	2.45	1.63
	20	3.19	2.12	1.92	1.28	3.53	2.35	2.17	1.45	4.08	2.71	2.56	1.70
	22	3.86	2.57	2.09	1.39	4.27	2.84	2.43	1.61	4.94	3.28	2.95	1.96
	24	4.60	3.06	2.37	1.58	5.08	3.38	2.76	1.84	5.88	3.91	3.37	2.24
	26	5.40	3.59	2.65	1.77	5.96	3.97	3.10	2.06	6.90	4.59	3.80	2.53
	28	6.26	4.16	2.94	1.95	6.92	4.60	3.44	2.29	8.00	5.32	4.24	2.82
	30	7.19	4.78	3.22	2.14	7.94	5.28	3.79	2.52	9.18	6.11	4.67	3.11
	32	8.18	5.44	3.50	2.33	9.03	6.01	4.13	2.75	10.4	6.95	5.11	3.40
Other Constants and Properties													
$b_y \times 10^3, (\text{kip-ft})^{-1}$		8.58	5.71	9.50	6.32	10.9	7.27						
$t_y \times 10^3, (\text{kips})^{-1}$		1.10	0.733	1.21	0.802	1.35	0.900						
$t_x \times 10^3, (\text{kips})^{-1}$		1.35	0.903	1.48	0.987	1.66	1.11						
$r_x/r_y$		5.03				4.98				5.02			
$r_y, \text{in.}$		1.99				1.98				1.95			

<sup>c</sup> Shape is slender for compression with  $F_y = 50 \text{ ksi}$ .

Note: Heavy line indicates  $KL/r_y$  equal to or greater than 200.

Effective length,  $KL$  (ft), with respect to least radius of gyration,  $r_y$ ,  
or Unbraced Length,  $L_b$  (ft), for X-X axis bending

Shape		W24 $\times$											
		103 <sup>c</sup>				94 <sup>c</sup>				84 <sup>c</sup>			
Design		$p \times 10^3$		$b_x \times 10^3$		$p \times 10^3$		$b_x \times 10^3$		$p \times 10^3$		$b_x \times 10^3$	
		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>		(kips) <sup>-1</sup>		(kip-ft) <sup>-1</sup>	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
least radius of gyration, $r_y$ , X-X axis bending	0	1.13	0.753	1.27	0.847	1.26	0.840	1.40	0.933	1.46	0.968	1.59	1.06
	11	1.52	1.01	1.42	0.944	1.67	1.11	1.57	1.05	1.92	1.28	1.80	1.20
	12	1.62	1.08	1.46	0.972	1.78	1.18	1.62	1.08	2.03	1.35	1.87	1.24
	13	1.73	1.15	1.51	1.00	1.90	1.26	1.68	1.12	2.17	1.44	1.93	1.28
	14	1.86	1.23	1.55	1.03	2.04	1.36	1.73	1.15	2.33	1.55	2.00	1.33
	15	2.00	1.33	1.61	1.07	2.21	1.47	1.79	1.19	2.52	1.68	2.08	1.38
	16	2.18	1.45	1.66	1.10	2.40	1.60	1.86	1.24	2.75	1.83	2.16	1.44
	17	2.38	1.58	1.72	1.14	2.62	1.74	1.93	1.28	3.01	2.00	2.25	1.49
	18	2.61	1.74	1.78	1.19	2.88	1.92	2.01	1.33	3.32	2.21	2.34	1.56
	19	2.88	1.92	1.85	1.23	3.18	2.12	2.09	1.39	3.68	2.45	2.45	1.63
	20	3.19	2.12	1.92	1.28	3.53	2.35	2.17	1.45	4.08	2.71	2.56	1.70
Other Constants and Properties													
$b_y \times 10^3$ , (kip-ft) <sup>-1</sup>		8.58		5.71		9.50		6.32		10.9		7.27	
$t_y \times 10^3$ , (kips) <sup>-1</sup>		1.10		0.733		1.21		0.802		1.35		0.900	
$t_r \times 10^3$ , (kips) <sup>-1</sup>		1.35		0.903		1.48		0.987		1.66		1.11	
$r_x/r_y$		5.03				4.98				5.02			
$r_y$ , in.		1.99				1.98				1.95			

<sup>c</sup> Shape is slender for compression with  $F_y = 50$  ksi.  
Note: Heavy line indicates  $KL/r_y$  equal to or greater than 200.

# Questions